7.6 Heat transfer during cross flow over cylinders

Fluid flow pattern

- Regimes of fluid flow across circular cylinders

- Vertex-shedding frequency, $f_v$

- Strouhal number, $St = \frac{f v}{\omega_o} = fn(Re_D)$

- Behind an object, the vortex-shedding frequency rises almost linearly with velocity

Heat transfer

- Giedt’s local measurements data

- Churchill and Bernstein’s correlation

\[
Nu_D = 0.3 + \frac{0.62 \, Re_D^{1/3} \, Pr^{1/3}}{1 + (0.4 / Pr)^{5/3}} \times \left[ 1 + \left( \frac{Re_D}{282000} \right)^{0.54} \right]
\]
**Example 7.7**

A electric resistance wire heater 0.0001 m in diameter is placed perpendicular to an air flow. $T_w = 40^\circ$C, $T_\infty = 20^\circ$C, $Q = 17.8 \text{ W/m}$.

- To find $u_\infty = ?$

  **Solution**

  \[
  \dot{Q} = \frac{Q}{\pi D(T_w - T_\infty)} = 2833 \text{ W/m}^2
  \]

  \[
  Nu = \frac{\dot{Q}}{k D} = 10.75
  \]

  \[
  Nu_\infty = 0.3 + \frac{0.62 Re_{0.01}^{0.5} Pr^{1/3}}{1 + (0.4 / Pr)^{1/3}}
  \]

  $Re_0 = 463, \quad u_\infty = \frac{v}{D} Re_0 = 73.9 \text{ m/s}$

---

**Heat transfer during flow across tube bundles**

- Aligned and staggered tube rows in tube bundles.
- Power-law relations

  \[
  Nu_\infty = C Re_0^{n} Pr^{1/3}
  \]

  Where

  \[
  Re_0 = \frac{\dot{u}_\infty D}{\nu}
  \]

  \[
  Nu_\infty = Pr^{2/3}\left(Pr / Pr_\infty\right) \frac{\dot{u}_\infty}{D} fn(Re_0)
  \]

  with $n = \begin{cases} 0 & \text{for gases} \\ 1/4 & \text{for liquids} \end{cases}$

  $fn(Re_0)$ see eq (7.71)

---

**8. Natural convection in single-phase fluids and during film condensation**

**8.1 Scope**

- Nature convection
  - Fluid flow are driven by body forces exerted directly on the fluid as a result of heating or cooling

- Two alike processes
  - Nature convection in a single-phase fluid
  - Film condensation

**8.2 The nature of film condensation and of natural convection**

- Nature convection
  - Vertical isothermal plate cools the fluid adjacent it

- Film condensation
Description

- Natural convection from a vertical heated plate.

A condensate film can only move downward.

Governing equations

- Outside the boundary layer

\[
\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= - \frac{1}{\rho} \frac{dp}{dx} + g + \frac{\partial^2 u}{\partial y^2} \\
\frac{dp}{dx} &= \rho g
\end{align*}
\]

- For nature convection

\[
\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= - \frac{1}{\rho} \frac{dp}{dx} + g + \frac{\partial^2 u}{\partial y^2} \\
\frac{\partial u}{\partial y} &= 0
\end{align*}
\]

- For film condensation

\[
\begin{align*}
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= - \frac{1}{\rho} \frac{dp}{dx} + g + \frac{\partial^2 u}{\partial y^2} \\
\frac{\partial u}{\partial y} &= 0
\end{align*}
\]

Energy equation

\[
\begin{align*}
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \alpha \frac{\partial^2 T}{\partial y^2}
\end{align*}
\]

8.3 Laminar natural convection on a vertical isothermal surface
Dimensional analysis and experimental data

- **Thermal expansion**
  \[
  \beta = \frac{1}{\nu} \left( \frac{RT}{P} \right) = \frac{1}{\nu} \frac{\rho \ddot{u}}{\dot{\rho} T} \approx \frac{1}{\nu} \frac{\rho - \rho_0}{\rho} \frac{1}{T - T_0} = \left( \frac{\rho_0}{\rho} \right) (T - T_0)
  \]

  - *Specific volume* 
  \[
  (1 - \rho_0 / \rho) g = -g \beta (T - T_0)
  \]

- *To eliminate \( \rho \) from eqs. in favor of \( T \)

Coupling of momentum and energy eqs. is very clearly

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -g \beta \left( T - T_0 \right) + \nu \frac{\partial^2 u}{\partial y^2}
\]

- *Coupling of momentum and energy eqs. is very clearly*

8. Natural convection in single-phase fluids and during film condensation

8.3 Laminar natural convection on a vertical isothermal surface

- **Dimensional functional eq.**

\[
\Pi_1 = \frac{Nu}{\beta} = \frac{\beta}{k} \quad \text{Nu}
\]

- 8-4-4 pi-groups

\[
\Pi_1 = Nu = \frac{\beta}{k} \quad \text{Nu}
\]

- *Nusselt number*

\[
\Pi_2 = Pr = \frac{\nu}{a} \quad \text{Prandtl number}
\]

\[
\Pi_3 = L \quad \text{Lamnkan}
\]

\[
\Pi_4 = \beta \quad \text{Thermal expansion}
\]

- *Buoyant force*

\[
\Pi_5 = \nu \quad \text{Viscous force}
\]

\[
\Pi_6 = \beta \left( T_0 - T_* \right) \quad \text{thermal expansion}
\]

- *Combination of buoyant and viscous force is very clearly*

8. Natural convection in single-phase fluids and during film condensation

8.3 Laminar natural convection on a vertical isothermal surface

- **Dimensional functional eq.**

\[
h = \text{fn}(Gr, Pr)
\]

\[
\text{Nu} = \text{fn}(Ra, Pr)
\]

- *Accuracy of a few percent*

- *Nu rises more sharply*

- *Data scatter increased*

- *Applicable to full range of Pr.*

h of natural convection on a vertical surface

- **Squire-Eckert (1930) formulation**

\[
\Theta = \frac{y}{\delta} \left( 1 + \frac{y}{\delta} \right) \quad \Theta = \frac{T - T_*}{T - T_*}
\]

- *Integral method*

- *Assume \( \delta \) is proportional to \( \rho \)

\[
\delta \left( \frac{\partial u}{\partial y} \right) dy = -v \left( \frac{\partial u}{\partial y} \right) dy + g \beta \left( T - T_0 \right) dy
\]

\[
\delta \left( \frac{\partial T}{\partial y} \right) dy = \frac{q_{\text{in}}}{\rho c_p} = -a \left( \frac{\partial T}{\partial y} \right) dy
\]

- *Estimation of profile*

\[
\Theta = 1 - \frac{y}{\delta} + \left( \frac{y}{\delta} \right)^2 \quad \Theta = \frac{T - T_*}{T - T_*}
\]

- b.c.'s

\[
\Theta = 1 \quad \text{at} \quad y / \delta = 0
\]

\[
\Theta = 0 \quad \text{at} \quad y / \delta = 1
\]

\[
\frac{d\Theta}{dy} = 0 \quad \text{at} \quad y / \delta = 1
\]

- *Find*

\[
\alpha = 1 \quad \beta = -2 \quad c = 1
\]

\[
\Theta = 1 - \frac{y}{\delta}
\]


**8. Natural convection in single-phase fluids and during film condensation**

### 8.3 Laminar natural convection on a vertical isothermal surface

#### h of natural convection on a vertical surface

- **Estimation of profile**
  
  \[ u = u(x) \left( \frac{y}{\delta} \right) + \frac{1}{v} \left( \frac{y}{\delta} \right) + \frac{1}{\nu \beta} \frac{d}{\delta} \left( \frac{y}{\delta} \right) \]

  \[ T \text{ is a profile for convection on a vertical isothermal surface} \]

- **b.c.’s**
  
  \[ u_{\delta/\nu} = 0 \quad u_{\delta/\nu} = 0 \]

  \[ \frac{dy}{dx} = 1 \]

- **find**
  
  \[ c = -2 \quad d = 1 \]

  \[ u = u(x) \frac{y}{\delta} \left( 1 - \frac{y}{\delta} \right) \]

**8.3 Laminar natural convection on a vertical isothermal surface**

#### h of natural convection on a vertical surface

- **C1 and C1** are determined by M and E integral eqs.
  
  \[ \int u \frac{dx}{T} \frac{dy}{T} = -v \left( \frac{\partial u}{\partial y} \right)_{y=0} + g \beta \int (T - T_0) dy \]

  \[ \text{becomes} \]

  \[ C_1 \left( \frac{\beta g (T - T_0)}{v} \right) \int \delta \left( \frac{y}{\delta} \right) \left( 1 - \frac{y}{\delta} \right) \frac{d}{\delta} \left( \frac{y}{\delta} \right) \]

  \[ = -C_1 \beta g (T - T_0) \beta \frac{\partial y}{\partial y} \int \delta \left( \frac{y}{\delta} \right) \left( 1 - \frac{y}{\delta} \right) \frac{d}{\delta} \left( \frac{y}{\delta} \right) + \beta g (T - T_0) \int \delta \left( \frac{y}{\delta} \right) \frac{d}{\delta} \left( \frac{y}{\delta} \right)_{y=0} \]

  \[ \delta = \frac{84}{3} \frac{1 - C_1}{C_1^2 \beta g (T - T_0) \beta} \]

  \[ \text{(8.20)} \]

**8.3 Laminar natural convection on a vertical isothermal surface**

#### h of natural convection on a vertical surface

- **Energy integral eq. becomes**
  
  \[ \frac{d}{dx} u(T - T_0) dy = \frac{\partial T}{\partial y} \]

  \[ (T - T_0) C_1 \frac{\beta g (T - T_0)}{v} \int \delta \left( \frac{y}{\delta} \right) \left( 1 - \frac{y}{\delta} \right) \frac{d}{\delta} \left( \frac{y}{\delta} \right) \]

  \[ = -C_1 \frac{\beta g (T - T_0)}{v} \int \delta \left( \frac{y}{\delta} \right) \left( 1 - \frac{y}{\delta} \right) \frac{d}{\delta} \left( \frac{y}{\delta} \right)_{y=0} \]

  \[ C_1 \frac{d}{dx} \left( \frac{2}{Pr} \frac{\beta g (T - T_0)}{v^2} \right) = \delta \]

  \[ \delta = \frac{80}{C_1 Pr \beta g (T - T_0)} \]

  \[ \text{(8.21)} \]
8. Natural convection in single-phase fluids and during film condensation

### 8.3 Laminar natural convection on a vertical isothermal surface

**Heat flux**

\[ q = \frac{\partial T}{\partial y} = -k \left( \frac{\partial T}{\partial y} \right) \delta \]

\[ Nu = \frac{qL}{k(T - T_w)} = 2 \frac{k(T - T_w)}{\Delta T k} = 2 \frac{k(T - T_w)}{\Delta T k} \]

\[ \text{Local Nu} \]

\[ \bar{h} = \frac{T - T_w}{T - T_{\infty}} = \frac{1}{k} \int_0^k \frac{\partial T}{\partial y} \delta \]

\[ \text{Mean Nu} \]

\[ \overline{Nu} = 0.678Ra^{1/4} \frac{Pr}{0.952 + Pr} \]

**Example 8.2**

- Large thin metal sheet at \( T_i \) dipped in a bath at \( T \).
- To find: the time for \( T_{\text{metal}} = T \).
- Solution

\[ \frac{d(T - T_{\infty})}{dt} = \frac{hA}{\rho c V} (T - T_{\infty}) \]

\[ (T - T_{\infty}) = e^{-\frac{t}{\bar{h} L^2 \frac{Pr}{0.952 + Pr}}} \]

\[ \bar{h} = 0.678 \frac{k}{L \left( \frac{Pr}{0.952 + Pr} \right)^{1/4}} \left( \frac{g \beta L}{\alpha V} \right)^{1/4} \Delta T^{1/4} = B \Delta T^{1/4} \]

**Comparison of analysis and correlations with experimental data**

- Churchill and Chu

\[ Nu_{\text{ Churchill and Chu}} = f_n(Ra, Pr) \]

**Variable-properties problem**

- Properties of the fluid
  - \( \beta \) should be evaluated at \( T_w \)
  - All other properties should be evaluated at \( T_i = T - C(T_i - T_{\infty}) \)
    - For gas: \( C = 0.38 \)

**Note on the validity of b.l. approximations**

- If \( \delta \) is fairly large, especially when Gr is small
- The b.l. assumptions are quite unreasonable
- Numerical solutions of full Navier-Stokes equations are necessary
- The form of the correlation \( Nu_x = f_n(Ra, Pr) \)
8. Natural convection in single-phase fluids and during film condensation

8.3 Laminar natural convection on a vertical isothermal surface

Homework

- 7.4
- 7.28
- 8.2
- 8.6